WING-BODY INTERACTION IN LINEAR SUPERSONIC FLOW

BY

ABRAHAM CHARNES

B.A., University of Illinois, 1938
M.S., University of Illinois, 1939

AN ABSTRACT OF A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS IN THE GRADUATE COLLEGE OF THE UNIVERSITY OF ILLINOIS, 1947

R. UNIV. BIBLIOTHEEK LEIDEN

URBANA, ILLINOIS
1952

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Let φ^W be the velocity potential of a steady incompressible irrotational supersonic flow about a wing at small angle of attack is the usual linear approximation. Let the flow velocity be in the positive x-direction (x,y,z rectangular cartesian coordinates) and constant up to the leading point of the wing. Let the surface of the wing be in $z=0$ as regards the boundary condition of no flow into the wing. Then for suitable choice of units φ^W satisfies the linearized differential equation

$$\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-\frac{\partial^2\varphi}{\partial x^2}=0 \tag{1}$$

plus

$$\varphi=0,\ \frac{\partial\varphi}{\partial x}=\text{the constant flow velocity on } x=0 \tag{1.1}$$

and

$$\frac{\partial\varphi}{\partial z}=0 \text{ on that part of } z=0 \text{ covered by the wing.} \tag{1.2}$$

Let an infinite circular cylinder of radius a be introduced about the x-axis; also cylindrical coordinates via $y=r\cos\theta$, $z=r\sin\theta$. If $\varphi=\varphi^W+\varphi^A$ is the velocity potential of flow about the wing-body combination, then φ^A satisfies (1), (1.2), $\varphi^A=\dfrac{\partial\varphi^A}{\partial x}=0$ on $x=0$, and

$$\frac{\partial\varphi^A}{\partial r}=-\frac{\partial\varphi^W}{\partial r} \text{ on } r=a. \tag{1.3}$$

Particular solutions of (1) in cylindrical coordinates which are exponential in x, trigonometric in θ are available for superposition. The boundary conditions suggest inverse Laplace transform representation on x, fourier series on θ. The form eventually obtained for φ^A most convenient for calculation of the quantities of interest in the applications, φ^A and $\dfrac{\varphi^A}{\partial\theta}$, is

$$\varphi^A(x,r,\theta)=-\frac{1}{2\pi}\int_0^x f_0(x-\xi,a)\,C_0\left(\frac{\xi}{a},\frac{r}{a}\right)d\xi$$
$$-\frac{1}{\pi}\sum_1^\infty \cos m\theta\int_0^x f_m(x-\xi,a)\,C_m\left(\frac{\xi}{a},\frac{r}{a}\right)d\xi \tag{2}$$

for $0\leqslant\theta\leqslant\pi$, where $C_m\left(\dfrac{\xi}{a},\dfrac{r}{a}\right)\equiv\dfrac{1}{2\pi i}\displaystyle\int_{B-i\infty}^{B+i\infty}\frac{K_m\left(p\frac{r}{a}\right)e^{\frac{x}{a}p}}{pK_m'(p)}dp$, f_m is

the m^{th} fourier coefficient of $\left.\frac{\partial \varphi^W(x-\xi,r,\theta)}{\partial r}\right|_{n=a}$ defined even on $(-\pi, \pi)$,

K_m is the m^{th} order modified Bessel function of the second kind.

A more convenient form of C_m for calculation is

$$(3)\quad C_m\left(\frac{\xi}{a},\frac{r}{a}\right) = -\sum_i \frac{z_i K_m\left(z_i\frac{r}{a}\right)e^{z_i\frac{x}{a}}}{(m^2+z_i^2)\,K_m(z_i)} + (-1)^m\int_0^\infty \frac{\left[I_m\left(\lambda\frac{r}{a}\right)K_m'(\lambda) - K_m\left(\lambda\frac{r}{a}\right)I_m'(\lambda)\right]e^{-\lambda\frac{x}{a}}}{[K_m'^2(\lambda)+\pi^2 I_m'^2(\lambda)]\,\lambda}\,d\lambda$$

where the z_i are roots of $K_m'(z)=0$ (these are finite in number, simple, have negative real parts and occur in conjugate complex pairs).

It is proved that φ^A is necessarily the unique solution where $f_m = 0\left(\frac{1}{m^2}\right)$ and the fourier coefficients g_m of $\frac{\partial}{\partial x}\left[\left.\frac{\partial \varphi^W(x-\xi,r,\theta)}{\partial r}\right|_{r=a}\right]$ defined even on $(-\pi, \pi)$ are $0\left(\frac{1}{m^{1+\delta}}\right)$, any $\delta>0$.

The technique of proof involves preliminary use of Plancherel and faltung theorems, piece-wise discussion of the integration range of majorizing integrals of combinations of Bessel functions and their derivatives by relating their functional properties to those of Nicholson's integral.

It is indicated how polygonal bodies may be treated by Volterra's method of images. The case of a rectangular body is solved by determining images and signs of the corresponding terms; only a finite number of images appear in the expression for the potential at a particular point.

Uniqueness of wing-body flows involving cylindrical bodies and wings whose leading edge may be reached from $x=0$ by characteristic cones which do not otherwise meet the wing is established by appropriate modification of a proof attributed to Zaremba (Courant-Hilbert, vol. 2, p. 379).

A corollary of the image method involving only one image is solution of wing-flows under the above restriction.

An extension of the image method, correlation of the values of the potential at pairs of points which are images with respect to a suitable plane through the field of flow, is introduced to determine and establish uniqueness in the case of the rectangular wing. It is shown that flows about certain wings of polygonal plan form too sharply raked back to conform to the previous restriction can be reduced step-by-step via Lorenz rotations to this case, hence are determinate and unique.

VITA

I was born September 4, 1917, in Hopewell, Virginia. I attended grade schools and Crane Technical High School in Chicago, Illinois. In September, 1934, I enrolled in the ceramics curriculum of the College of Engineering of the University of Illinois, changed to mathematics in February, 1937, was graduated B.A. June, 1938, Phi Beta Kappa and with highest honors in mathematics.

In June, 1939, I received the M.S. degree; in 1939-1940 and in 1940-1941, I was a University Fellow in mathematics. My chief interest was in the theory of operators as developed in the courses of D. G. Bourgin. Other course work was under P. W. Ketchum, R. Baer, H. Levy, A. B. Coble, H. R. Brahana in mathematics, H. M. Mott-Smith, R. Serber in physics, W. H. Rodebush in physical chemistry. During this time I took Ph.D. preliminary examinations and began work under D. G. Bourgin on a thesis in the field of linear topological spaces.

In view of world conditions, I attended the special national defense summer session at Brown University the summer of 1941 and followed this with a semester at the Graduate School of Engineering of Harvard University from which H. M. Mott-Smith engaged me for research in mines at the Bureau of Ordnance, Navy Department. I entered the Naval Reserve in May, 1943, as a volunteer specialist.

In the Navy I served chiefly in operational analysis and statistics of torpedo performance and design. During the latter months of the War I was loaned to A. P. L. for work on supersonic flow problems in N. Edmonson's theoretical section.